DARK INTEGERS

by Greg Egan

Our new story from Greg Egan is a stand-alone tale that follows on the events of "Luminous" (September 1995). It's also the first story we've seen from Greg since "Oracle" appeared in our July 2000 issue. The author tells us, though, that "after spending a few years away from writing, trying to assist some of the asylum seekers that Australia imprisons in remote detention centers, I recently completed my seventh SF novel, Incandescence, which is due to be published by Gollancz in the UK in May 2008." We hope that this return to writing means we'll soon be seeing more of his brilliant fiction.

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"Good morning, Bruno. How is the weather there in Sparseland?"

The screen icon for my interlocutor was a three-holed torus tiled with triangles, endlessly turning itself inside out. The polished tones of the male synthetic voice I heard conveyed no specific origin, but gave a sense nonetheless that the speaker's first language was something other than English.

I glanced out the window of my home office, taking in a patch of blue sky and the verdant gardens of a shady West Ryde cul-de-sac. Sam used "good morning" regardless of the hour, but it really was just after ten A.M., and the tranquil Sydney suburb was awash in sunshine and birdsong.

"Perfect," I replied. "I wish I wasn't chained to this desk."

There was a long pause, and I wondered if the translator had mangled the idiom, creating the impression that I had been shackled by ruthless assailants, who had nonetheless left me with easy access to my instant messaging program. Then Sam said, "I'm glad you didn't go for a run today. I've already tried Alison and Yuen, and they were both unavailable. If I hadn't been able to get through to you, it might have been difficult to keep some of my colleagues in check."

I felt a surge of anxiety, mixed with resentment. I refused to wear an iWatch, to make myself reachable twenty-four hours a day. I was a mathematician, not an obstetrician. Perhaps I was an amateur diplomat as well, but even if Alison, Yuen, and I didn't quite cover the time zones, it would never be more than a few hours before Sam could get hold of at

least one of us.

“I didn’t realize you were surrounded by hotheads,” I replied. “What’s the great emergency?” I hoped the translator would do justice to the sharpness in my voice. Sam’s colleagues were the ones with all the firepower, all the resources; they should not have been jumping at shadows. True, we had once tried to wipe them out, but that had been a perfectly innocent mistake, more than ten years before.

Sam said, “Someone from your side seems to have jumped the border.”

“*Jumped* it?”

“As far as we can see, there’s no trench cutting through it. But a few hours ago, a cluster of propositions on our side started obeying your axioms.”

I was stunned. “An isolated cluster? With no derivation leading back to us?”

“None that we could find.”

I thought for a while. “Maybe it was a natural event. A brief surge across the border from the background noise that left a kind of tidal pool behind.”

Sam was dismissive. “The cluster was too big for that. The probability would be vanishingly small.” Numbers came through on the data channel; he was right.

I rubbed my eyelids with my fingertips; I suddenly felt very tired. I’d thought our old nemesis, Industrial Algebra, had given up the chase long ago. They had stopped offering bribes and sending mercenaries to harass me, so I’d assumed they’d finally written off the defect as a hoax or a mirage, and gone back to their core business of helping the world’s military kill and maim people in ever more technologically sophisticated ways.

Maybe this wasn’t IA. Alison and I had first located the defect—a set of contradictory results in arithmetic that marked the border between our mathematics and the version underlying Sam’s world—by means of a vast set of calculations farmed out over the internet, with thousands of volunteers donating their computers’ processing power when the machines would otherwise have been idle. When we’d pulled the plug on that

project—keeping our discovery secret, lest IA find a way to weaponize it—a few participants had been resentful, and had talked about continuing the search. It would have been easy enough for them to write their own software, adapting the same open source framework that Alison and I had used, but it was difficult to see how they could have gathered enough supporters without launching some kind of public appeal.

I said, “I can’t offer you an immediate explanation for this. All I can do is promise to investigate.”

“I understand,” Sam replied.

“You have no clues yourself ?” A decade before, in Shanghai, when Alison, Yuen, and I had used the supercomputer called Luminous to mount a sustained attack on the defect, the mathematicians of the far side had grasped the details of our unwitting assault clearly enough to send a plume of alternative mathematics back across the border with pinpoint precision, striking at just the three of us.

Sam said, “If the cluster had been connected to something, we could have followed the trail. But in isolation it tells us nothing. That’s why my colleagues are so anxious.”

“Yeah.” I was still hoping that the whole thing might turn out to be a glitch—the mathematical equivalent of a flock of birds with a radar echo that just happened to look like something more sinister—but the full gravity of the situation was finally dawning on me.

The inhabitants of the far side were as peaceable as anyone might reasonably wish their neighbors to be, but if their mathematical infrastructure came under threat they faced the real prospect of annihilation. They had defended themselves from such a threat once before, but because they had been able to trace it to its source and understand its nature, they had shown great forbearance. They had not struck their assailants dead, or wiped out Shanghai, or pulled the ground out from under our universe.

This new assault had not been sustained, but nobody knew its origins, or what it might portend. I believed that our neighbors would do no more than they had to in order to ensure their survival, but if they were forced to strike back blindly, they might find themselves with no path to safety short of turning our world to dust.

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Shanghai time was only two hours behind Sydney, but Yuen's IM status was still "unavailable." I emailed him, along with Alison, though it was the middle of the night in Zurich and she was unlikely to be awake for another four or five hours. All of us had programs that connected us to Sam by monitoring, and modifying, small portions of the defect: altering a handful of precariously balanced truths of arithmetic, wiggling the border between the two systems back and forth to encode each transmitted bit. The three of us on the near side might have communicated with each other in the same way, but on consideration we'd decided that conventional cryptography was a safer way to conceal our secret. The mere fact that communications data seemed to come from nowhere had the potential to attract suspicion, so we'd gone so far as to write software to send fake packets across the net to cover for our otherwise inexplicable conversations with Sam; anyone but the most diligent and resourceful of eavesdroppers would conclude that he was addressing us from an internet café in Lithuania.

While I was waiting for Yuen to reply, I scoured the logs where my knowledge miner deposited results of marginal relevance, wondering if some flaw in the criteria I'd given it might have left me with a blind spot. If anyone, anywhere had announced their intention to carry out some kind of calculation that might have led them to the defect, the news should have been plastered across my desktop in flashing red letters within seconds. Granted, most organizations with the necessary computing resources were secretive by nature, but they were also unlikely to be motivated to indulge in such a crazy stunt. Luminous itself had been decommissioned in 2012; in principle, various national security agencies, and even a few IT-centric businesses, now had enough silicon to hunt down the defect if they'd really set their sights on it, but as far as I knew Yuen, Alison, and I were still the only three people in the world who were certain of its existence. The black budgets of even the most profligate governments, the deep pockets of even the richest tycoons, would not stretch far enough to take on the search as a long shot, or an act of whimsy.

An IM window popped up with Alison's face. She looked ragged. "What time is it there?" I asked.

"Early. Laura's got colic."

"Ah. Are you okay to talk?"

"Yeah, she's asleep now."

My email had been brief, so I filled her in on the details. She pondered the matter in silence for a while, yawning unashamedly.

“The only thing I can think of is some gossip I heard at a conference in Rome a couple of months ago. It was a fourth-hand story about some guy in New Zealand who thinks he’s found a way to test fundamental laws of physics by doing computations in number theory.”

“Just random crackpot stuff, or ... what?”

Alison massaged her temples, as if trying to get more blood flowing to her brain. “I don’t know, what I heard was too vague to make a judgment. I gather he hasn’t tried to publish this anywhere, or even mentioned it in blogs. I guess he just confided in a few people directly, one of whom must have found it too amusing for them to keep their mouth shut.”

“Have you got a name?”

She went off camera and rummaged for a while. “Tim Campbell,” she announced. Her notes came through on the data channel. “He’s done respectable work in combinatorics, algorithmic complexity, optimization. I scoured the net, and there was no mention of this weird stuff. I was meaning to email him, but I never got around to it.”

I could understand why; that would have been about the time Laura was born. I said, “I’m glad you still go to so many conferences in the flesh. It’s easier in Europe, everything’s so close.”

“Ha! Don’t count on it continuing, Bruno. You might have to put your fat arse on a plane sometime yourself.”

“What about Yuen?”

Alison frowned. “Didn’t I tell you? He’s been in hospital for a couple of days. Pneumonia. I spoke to his daughter, he’s not in great shape.”

“I’m sorry.” Alison was much closer to him than I was; he’d been her doctoral supervisor, so she’d known him long before the events that had bound the three of us together.

Yuen was almost eighty. That wasn’t yet ancient for a middle-class Chinese man who could afford good medical care, but he would not be around forever.

I said, “Are we crazy, trying to do this ourselves?” She knew what I meant: liaising with Sam, managing the border, trying to keep the two worlds talking but the two sides separate, safe and intact.

Alison replied, “Which government would you trust not to screw this up? Not to try to exploit it?”

“None. But what’s the alternative? You pass the job on to Laura? Kate’s not interested in having kids. So do I pick some young mathematician at random to anoint as my successor?”

“Not at random, I’d hope.”

“You want me to advertise? ‘Must be proficient in number theory, familiar with Machiavelli, and own the complete boxed set of *The West Wing*?’”

She shrugged. “When the time comes, find someone competent you can trust. It’s a balance: the fewer people who know, the better, so long as there are always enough of us that the knowledge doesn’t risk getting lost completely.”

“And this goes on generation after generation? Like some secret society? The Knights of the Arithmetic Inconsistency?”

“I’ll work on the crest.”

We needed a better plan, but this wasn’t the time to argue about it. I said, “I’ll contact this guy Campbell and let you know how it goes.”

“Okay. Good luck.” Her eyelids were starting to droop.

“Take care of yourself.”

Alison managed an exhausted smile. “Are you saying that because you give a damn, or because you don’t want to end up guarding the Grail all by yourself ?”

“Both, of course.”

* * * *

“I have to fly to Wellington tomorrow.”

Kate put down the pasta-laden fork she'd raised halfway to her lips and gave me a puzzled frown. "That's short notice."

"Yeah, it's a pain. It's for the Bank of New Zealand. I have to do something on-site with a secure machine, one they won't let anyone access over the net."

Her frown deepened. "When will you be back?"

"I'm not sure. It might not be until Monday. I can probably do most of the work tomorrow, but there are certain things they restrict to the weekends, when the branches are off-line. I don't know if it will come to that."

I hated lying to her, but I'd grown accustomed to it. When we'd met, just a year after Shanghai, I could still feel the scar on my arm where one of Industrial Algebra's hired thugs had tried to carve a data cache out of my body. At some point, as our relationship deepened, I'd made up my mind that however close we became, however much I trusted her, it would be safer for Kate if she never knew anything about the defect.

"They can't hire someone local?" she suggested. I didn't think she was suspicious, but she was definitely annoyed. She worked long hours at the hospital, and she only had every second weekend off; this would be one of them. We'd made no specific plans, but it was part of our routine to spend this time together.

I said, "I'm sure they could, but it'd be hard to find someone at short notice. And I can't tell them to shove it, or I'll lose the whole contract. It's one weekend, it's not the end of the world."

"No, it's not the end of the world." She finally lifted her fork again.

"Is the sauce okay?"

"It's delicious, Bruno." Her tone made it clear that no amount of culinary effort would have been enough to compensate, so I might as well not have bothered.

I watched her eat with a strange knot growing in my stomach. Was this how spies felt, when they lied to their families about their work? But my own secret sounded more like something from a psychiatric ward. I was entrusted with the smooth operation of a treaty that I, and two friends, had struck with an invisible ghost world that coexisted with our own. The ghost

world was far from hostile, but the treaty was the most important in human history, because either side had the power to annihilate the other so thoroughly that it would make a nuclear holocaust seem like a pinprick.

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Victoria University was in a hilltop suburb overlooking Wellington. I caught a cable car, and arrived just in time for the Friday afternoon seminar. Contriving an invitation to deliver a paper here myself would have been difficult, but wangling permission to sit in as part of the audience was easy; although I hadn't been an academic for almost twenty years, my ancient Ph.D and a trickle of publications, however tenuously related to the topic of the seminar, were still enough to make me welcome.

I'd taken a gamble that Campbell would attend—the topic was peripheral to his own research, official or otherwise—so I was relieved to spot him in the audience, recognizing him from a photo on the faculty web site. I'd emailed him straight after I'd spoken to Alison, but his reply had been a polite brush-off: he acknowledged that the work I'd heard about on the grapevine owed something to the infamous search that Alison and I had launched, but he wasn't ready to make his own approach public.

I sat through an hour on "Monoids and Control Theory," trying to pay enough attention that I wouldn't make a fool of myself if the seminar organizer quizzed me later on why I'd been sufficiently attracted to the topic to interrupt my "sightseeing holiday" in order to attend. When the seminar ended, the audience split into two streams: one heading out of the building, the other moving into an adjoining room where refreshments were on offer. I saw Campbell making for the open air, and it was all I could do to contrive to get close enough to call out to him without making a spectacle.

"Dr. Campbell?"

He turned and scanned the room, probably expecting to see one of his students wanting to beg for an extension on an assignment. I raised a hand and approached him.

"Bruno Costanzo. I emailed you yesterday."

"Of course." Campbell was a thin, pale man in his early thirties. He shook my hand, but he was obviously taken aback. "You didn't mention that you were in Wellington."

I made a dismissive gesture. "I was going to, but then it seemed a bit

presumptuous.” I didn’t spell it out, I just left him to conclude that I was as ambivalent about this whole inconsistency nonsense as he was.

If fate had brought us together, though, wouldn’t it be absurd not to make the most of it?

“I was going to grab some of those famous scones,” I said; the seminar announcement on the web had made big promises for them. “Are you busy?”

“Umm. Just paperwork. I suppose I can put it off.”

As we made our way into the tea room, I waffled on airily about my holiday plans. I’d never actually been to New Zealand before, so I made it clear that most of my itinerary still lay in the future. Campbell was no more interested in the local geography and wildlife than I was; the more I enthused, the more distant his gaze became. Once it was apparent that he wasn’t going to cross-examine me on the finer points of various hiking trails, I grabbed a buttered scone and switched subject abruptly.

“The thing is, I heard you’d devised a more efficient strategy for searching for a defect.” I only just managed to stop myself from using the definite article; it was a while since I’d spoken about it as if it were still hypothetical. “You know the kind of computing power that Dr. Tierney and I had to scrounge up?”

“Of course. I was just an undergraduate, but I heard about the search.”

“Were you one of our volunteers?” I’d checked the records, and he wasn’t listed, but people had had the option of registering anonymously.

“No. The idea didn’t really grab me, at the time.” As he spoke, he seemed more discomfited than the failure to donate his own resources twelve years ago really warranted. I was beginning to suspect that he’d actually been one of the people who’d found the whole tongue-in-cheek conjecture that Alison and I had put forward to be unforgivably foolish. We had never asked to be taken seriously—and we had even put prominent links to all the worthy biomedical computing projects on our web page, so that people knew there were far better ways to spend their spare megaflops—but nonetheless, some mathematical/philosophical stuffed shirts had spluttered with rage at the sheer impertinence and naivety of our hypothesis. Before things turned serious, it was the entertainment value of that backlash that had made our efforts worthwhile.

“But now you’ve refined it somehow?” I prompted him, doing my best to let him see that I felt no resentment at the prospect of being outdone. In fact, the hypothesis itself had been Alison’s, so even if there hadn’t been more important things than my ego at stake, that really wasn’t a factor. As for the search algorithm, I’d cobbled it together on a Sunday afternoon, as a joke, to call Alison’s bluff. Instead, she’d called mine, and insisted that we release it to the world.

Campbell glanced around to see who was in earshot, but then perhaps it dawned on him that if the news of his ideas had already reached Sydney via Rome and Zurich, the battle to keep his reputation pristine in Wellington was probably lost.

He said, “What you and Dr. Tierney suggested was that random processes in the early universe might have included proofs of mutually contradictory theorems about the integers, the idea being that no computation to expose the inconsistency had yet had time to occur. Is that a fair summary?”

“Sure.”

“One problem I have with that is, I don’t see how it could lead to an inconsistency that could be detected here and now. If the physical system A proved theorem A, and the physical system B proved theorem B, then you might have different regions of the universe obeying different axioms, but it’s not as if there’s some universal mathematics textbook hovering around outside spacetime, listing every theorem that’s ever been proved, which our computers then consult in order to decide how to behave. The behavior of a classical system is determined by its own particular causal past. If we’re the descendants of a patch of the universe that proved theorem A, our computers should be perfectly capable of *disproving* theorem B, whatever happened somewhere else fourteen billion years ago.”

I nodded thoughtfully. “I can see what you’re getting at.” If you weren’t going to accept full-blooded Platonism, in which there *was* a kind of ghostly textbook listing the eternal truths of mathematics, then a half-baked version where the book started out empty and was only filled in line-by-line as various theorems were tested seemed like the worst kind of compromise. In fact, when the far side had granted Yuen, Alison, and I insight into their mathematics for a few minutes in Shanghai, Yuen had proclaimed that the flow of mathematical information *did* obey Einstein locality; there was no universal book of truths, just records of the past sloshing around at

lightspeed or less, intermingling and competing.

I could hardly tell Campbell, though, that not only did I know for a fact that a single computer could prove both a theorem and its negation, but depending on the order in which it attacked the calculations it could sometimes even shift the boundary where one set of axioms failed and the other took over.

I said, “And yet you still believe it’s worth searching for an inconsistency?”

“I do,” he conceded. “Though I came to the idea from a very different approach.” He hesitated, then picked up a scone from the table beside us.

“One rock, one apple, one scone. We have a clear idea of what we mean by those phrases, though each one might encompass ten-to-the-ten-to-the-thirty-something slightly different configurations of matter. My ‘one scone’ is not the same as your ‘one scone.’”

“Right.”

“You know how banks count large quantities of cash?”

“By weighing them?” In fact there were several other cross-checks as well, but I could see where he was heading and I didn’t want to distract him with nit-picking.

“Exactly. Suppose we tried to count scones the same way: weigh the batch, divide by some nominal value, then round to the nearest integer. The weight of any individual scone varies so much that you could easily end up with a version of arithmetic different from our own. If you ‘counted’ two separate batches, then merged them and ‘counted’ them together, there’s no guarantee that the result would agree with the ordinary process of integer addition.”

I said, “Clearly not. But digital computers don’t run on scones, and they don’t count bits by weighing them.”

“Bear with me,” Campbell replied. “It isn’t a perfect analogy, but I’m not as crazy as I sound. Suppose, now, that *everything* we talk about as ‘one thing’ has a vast number of possible configurations that we’re either ignoring deliberately, or are literally incapable of distinguishing. Even something as simple as an electron prepared in a certain quantum state.”

I said, “You’re talking about hidden variables now?”

“Of a kind, yes. Do you know about Gerard ‘t Hooft’s models for deterministic quantum mechanics?”

“Only vaguely,” I admitted.

“He postulated fully deterministic degrees of freedom at the Planck scale, with quantum states corresponding to equivalence classes containing many different possible configurations. What’s more, all the ordinary quantum states we prepare at an atomic level would be complex superpositions of those primordial states, which allows him to get around the Bell inequalities.” I frowned slightly; I more or less got the picture, but I’d need to go away and read ‘t Hooft’s papers.

Campbell said, “In a sense, the detailed physics isn’t all that important, so long as you accept that ‘one thing’ might not *ever* be exactly the same as another ‘one thing,’ regardless of the kind of objects we’re talking about. Given that supposition, physical processes that *seem* to be rigorously equivalent to various arithmetic operations can turn out not to be as reliable as you’d think. With scone-weighing, the flaws are obvious, but I’m talking about the potentially subtler results of misunderstanding the fundamental nature of matter.”

“Hmm.” Though it was unlikely that anyone else Campbell had confided in had taken these speculations as seriously as I did, not only did I not want to seem a pushover, I honestly had no idea whether anything he was saying bore the slightest connection to reality.

I said, “It’s an interesting idea, but I still don’t see how it could speed up the hunt for inconsistencies.”

“I have a set of models,” he said, “which are constrained by the need to agree with some of ‘t Hooft’s ideas about the physics, and also by the need to make arithmetic *almost* consistent for a very large range of objects. From neutrinos to clusters of galaxies, basic arithmetic involving the kinds of numbers we might encounter in ordinary situations should work out in the usual way.” He laughed. “I mean, that’s the world we’re living in, right?”

Some of us. “Yeah.”

“But the interesting thing is, I can’t make the physics work at all if the arithmetic doesn’t run askew eventually—if there aren’t trans-astronomical

numbers where the physical representations no longer capture the arithmetic perfectly. And each of my models lets me predict, more or less, where those effects should begin to show up. By starting with the fundamental physical laws, I can deduce a sequence of calculations with large integers that ought to reveal an inconsistency, when performed with pretty much any computer."

"Taking you straight to the defect, with no need to search at all." I'd let the definite article slip out, but it hardly seemed to matter anymore.

"That's the theory." Campbell actually blushed slightly. "Well, when you say 'no search,' what's involved really is a much smaller search. There are still free parameters in my models; there are potentially billions of possibilities to test."

I grinned broadly, wondering if my expression looked as fake as it felt. "But no luck yet?"

"No." He was beginning to become self-conscious again, glancing around to see who might be listening.

Was he lying to me? Keeping his results secret until he could verify them a million more times, and then decide how best to explain them to incredulous colleagues and an uncomprehending world? Or had whatever he'd done that had lobbed a small grenade into Sam's universe somehow registered in Campbell's own computer as arithmetic as usual, betraying no evidence of the boundary he'd crossed? After all, the offending cluster of propositions had obeyed *our* axioms, so perhaps Campbell had managed to force them to do so without ever realizing that they hadn't in the past. His ideas were obviously close to the mark—and I could no longer believe this was just a coincidence—but he seemed to have no room in his theory for something that I knew for a fact: arithmetic wasn't merely inconsistent, it was *dynamic*. You could take its contradictions and slide them around like bumps in a carpet.

Campbell said, "Parts of the process aren't easy to automate; there's some manual work to be done setting up the search for each broad class of models. I've only been doing this in my spare time, so it could be a while before I get around to examining all the possibilities."

"I see." If all of his calculations so far had produced just one hit on the far side, it was conceivable that the rest would pass without incident. He would publish a negative result ruling out an obscure class of physical theories, and life would go on as normal on both sides of the inconsistency.

What kind of weapons inspector would I be, though, to put my faith in that rosy supposition?

Campbell was looking fidgety, as if his administrative obligations were beckoning. I said, “It’d be great to talk about this a bit more while we’ve got the chance. Are you busy tonight? I’m staying at a backpacker’s down in the city, but maybe you could recommend a restaurant around here somewhere?”

He looked dubious for a moment, but then an instinctive sense of hospitality seemed to overcome his reservations. He said, “Let me check with my wife. We’re not really into restaurants, but I was cooking tonight anyway, and you’d be welcome to join us.”

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Campbell’s house was a fifteen minute walk from the campus; at my request, we detoured to a liquor store so I could buy a couple of bottles of wine to accompany the meal. As I entered the house, my hand lingered on the doorframe, depositing a small device that would assist me if I needed to make an uninvited entry in the future.

Campbell’s wife, Bridget, was an organic chemist, who also taught at Victoria University. The conversation over dinner was all about department heads, budgets, and grant applications, and, despite having left academia long ago, I had no trouble relating sympathetically to the couple’s gripes. My hosts ensured that my wine glass never stayed empty for long.

When we’d finished eating, Bridget excused herself to make a call to her mother, who lived in a small town on the south island. Campbell led me into his study and switched on a laptop with fading keys that must have been twenty years old. Many households had a computer like this: the machine that could no longer run the latest trendy bloatware, but which still worked perfectly with its original OS.

Campbell turned his back to me as he typed his password, and I was careful not to be seen even trying to look. Then he opened some C++ files in an editor, and scrolled over parts of his search algorithm.

I felt giddy, and it wasn’t the wine; I’d filled my stomach with an over-the-counter sobriety aid that turned ethanol into glucose and water faster than any human being could imbibe it. I fervently hoped that Industrial Algebra really had given up their pursuit; if I could get this close to

Campbell's secrets in half a day, IA could be playing the stock market with alternative arithmetic before the month was out, and peddling inconsistency weapons to the Pentagon soon after.

I did not have a photographic memory, and Campbell was just showing me fragments anyway. I didn't think he was deliberately taunting me; he just wanted me to see that he had something concrete, that all his claims about Planck scale physics and directed search strategies had been more than hot air.

I said, "Wait! What's that?" He stopped hitting the PAGE DOWN key, and I pointed at a list of variable declarations in the middle of the screen:

long int i1, i2, i3;

dark d1, d2, d3;

A "long int" was a long integer, a quantity represented by twice as many bits as usual. On this vintage machine, that was likely to be a total of just sixty-four bits. "What the fuck is a 'dark'?" I demanded. It wasn't how I'd normally speak to someone I'd only just met, but then, I wasn't meant to be sober.

Campbell laughed. "A dark integer. It's a type I defined. It holds four thousand and ninety-six bits."

"But why the name?"

"Dark matter, dark energy ... dark integers. They're all around us, but we don't usually see them, because they don't quite play by the rules."

Hairs rose on the back of my neck. I could not have described the infrastructure of Sam's world more concisely myself.

Campbell shut down the laptop. I'd been looking for an opportunity to handle the machine, however briefly, without arousing his suspicion, but that clearly wasn't going to happen, so as we walked out of the study I went for plan B.

"I'm feeling kind of..." I sat down abruptly on the floor of the hallway. After a moment, I fished my phone out of my pocket and held it up to him. "Would you mind calling me a taxi?"

"Yeah, sure." He accepted the phone, and I cradled my head in my

arms. Before he could dial the number, I started moaning softly. There was a long pause; he was probably weighing up the embarrassment factor of various alternatives.

Finally he said, “You can sleep here on the couch if you like.” I felt a genuine pang of sympathy for him; if some clown I barely knew had pulled a stunt like this on me, I would at least have made him promise to foot the cleaning bills if he threw up in the middle of the night.

In the middle of the night, I did make a trip to the bathroom, but I kept the sound effects restrained. Halfway through, I walked quietly to the study, crossed the room in the dark, and slapped a thin, transparent patch over the adhesive label that a service company had placed on the outside of the laptop years before. My addition would be invisible to the naked eye, and it would take a scalpel to prise it off. The relay that would communicate with the patch was larger, about the size of a coat button; I stuck it behind a bookshelf. Unless Campbell was planning to paint the room or put in new carpet, it would probably remain undetected for a couple of years, and I’d already prepaid a two year account with a local wireless internet provider.

I woke not long after dawn, but this un-Bacchanalian early rising was no risk to my cover; Campbell had left the curtains open so the full force of the morning sun struck me in the face, a result that was almost certainly deliberate. I tiptoed around the house for ten minutes or so, not wanting to seem too organized if anyone was listening, then left a scrawled note of thanks and apology on the coffee table by the couch, before letting myself out and heading for the cable car stop.

Down in the city, I sat in a café opposite the backpacker’s hostel and connected to the relay, which in turn had established a successful link with the polymer circuitry of the laptop patch. When noon came and went without Campbell logging on, I sent a message to Kate telling her that I was stuck in the bank for at least another day.

I passed the time browsing the news feeds and buying overpriced snacks; half of the café’s other patrons were doing the same. Finally, just after three o’clock, Campbell started up the laptop.

The patch couldn’t read his disk drive, but it could pick up currents flowing to and from the keyboard and the display, allowing it to deduce everything he typed and everything he saw. Capturing his password was easy. Better yet, once he was logged in he set about editing one of his files, extending his search program to a new class of models. As he scrolled back and forth, it wasn’t long before the patch’s screen shots

encompassed the entire contents of the file he was working on.

He labored for more than two hours, debugging what he'd written, then set the program running. This creaky old twentieth century machine, which predated the whole internet-wide search for the defect, had already scored one direct hit on the far side; I just hoped this new class of models were all incompatible with the successful ones from a few days before.

Shortly afterward, the IR sensor in the patch told me that Campbell had left the room. The patch could induce currents in the keyboard connection; I could type into the machine as if I was right there. I started a new process window. The laptop wasn't connected to the internet at all, except through my spyware, but it took me only fifteen minutes to display and record everything there was to see: a few library and header files that the main program depended on, and the data logs listing all of the searches so far. It would not have been hard to hack into the operating system and make provisions to corrupt any future searches, but I decided to wait until I had a better grasp of the whole situation. Even once I was back in Sydney, I'd be able to eavesdrop whenever the laptop was in use, and intervene whenever it was left unattended. I'd only stayed in Wellington in case there'd been a need to return to Campbell's house in person.

When evening fell and I found myself with nothing urgent left to do, I didn't call Kate; it seemed wiser to let her assume that I was slaving away in a windowless computer room. I left the café and lay on my bed in the hostel. The dormitory was deserted; everyone else was out on the town.

I called Alison in Zurich and brought her up to date. In the background, I could hear her husband, Philippe, trying to comfort Laura in another room, calmly talking baby-talk in French while his daughter wailed her head off.

Alison was intrigued. "Campbell's theory can't be perfect, but it must be close. Maybe we'll be able to find a way to make it fit in with the dynamics we've seen." In the ten years since we'd stumbled on the defect, all our work on it had remained frustratingly empirical: running calculations and observing their effects. We'd never come close to finding any deep underlying principles.

"Do you think Sam knows all this?" she asked.

"I have no idea. If he did, I doubt he'd admit it." Though it was Sam who had given us a taste of far-side mathematics in Shanghai, that had really just been a clip over the ear to let us know that what we were trying to

wipe out with Luminous was a civilization, not a wasteland. After that near-disastrous first encounter, he had worked to establish communications with us, learning our languages and happily listening to the accounts we'd volunteered of our world, but he had not been equally forthcoming in return. We knew next to nothing about far-side physics, astronomy, biology, history, or culture. That there were living beings occupying the same space as the Earth suggested that the two universes were intimately coupled somehow, in spite of their mutual invisibility. But Sam had hinted that life was much more common on his side of the border than ours; when I'd told him that we seemed to be alone, at least in the solar system, and were surrounded by light-years of sterile vacuum, he'd taken to referring to our side as "Sparseland."

Alison said, "Either way, I think we should keep it to ourselves. The treaty says we should do everything in our power to deal with any breach of territory of which the other side informs us. We're doing that. But we're not obliged to disclose the details of Campbell's activities."

"That's true." I wasn't entirely happy with her suggestion, though. In spite of the attitude Sam and his colleagues had taken—in which they assumed that anything they told us might be exploited, might make them more vulnerable—a part of me had always wondered if there was some gesture of good faith we could make, some way to build trust. Since talking to Campbell, in the back of my mind I'd been building up a faint hope that his discovery might lead to an opportunity to prove, once and for all, that our intentions were honorable.

Alison read my mood. She said, "Bruno, they've given us *nothing*. Shanghai excuses a certain amount of caution, but we also know from Shanghai that they could brush Luminous aside like a gnat. They have enough computing power to crush us in an instant, and they still cling to every strategic advantage they can get. Not to do the same ourselves would just be stupid and irresponsible."

"So you want us to hold on to this secret weapon?" I was beginning to develop a piercing headache. My usual way of dealing with the surreal responsibility that had fallen on the three of us was to pretend that it didn't exist; having to think about it constantly for three days straight meant more tension than I'd faced for a decade. "Is that what it's come down to? Our own version of the Cold War? Why don't you just march into NATO headquarters on Monday and hand over everything we know?"

Alison said dryly, "Switzerland isn't a member of NATO. The government here would probably charge me with treason."

I didn't want to fight with her. "We should talk about this later. We don't even know exactly what we've got. I need to go through Campbell's files and confirm whether he really did what we think he did."

"Okay."

"I'll call you from Sydney."

It took me a while to make sense of everything I'd stolen from Campbell, but eventually I was able to determine which calculations he'd performed on each occasion recorded in his log files. Then I compared the propositions that he'd tested with a rough, static map of the defect; since the event Sam had reported had been deep within the far side, there was no need to take account of the small fluctuations that the border underwent over time.

If my analysis was correct, late on Wednesday night Campbell's calculations had landed in the middle of far-side mathematics. He'd been telling me the truth, though; he'd found nothing out of the ordinary there. Instead, the thing he had been seeking had melted away before his gaze.

In all the calculations Alison and I had done, only at the border had we been able to force propositions to change their allegiance and obey our axioms. It was as if Campbell had dived in from some higher dimension, carrying a hosepipe that sprayed everything with the arithmetic we knew and loved.

For Sam and his colleagues, this was the equivalent of a suitcase nuke appearing out of nowhere, as opposed to the ICBMs they knew how to track and annihilate. Now Alison wanted us to tell them, "Trust us, we've dealt with it," without showing them the weapon itself, without letting them see how it worked, without giving them a chance to devise new defenses against it.

She wanted us to have something up our sleeves, in case the hawks took over the far side, and decided that Sparseland was a ghost world whose lingering, baleful presence they could do without.

Drunken Saturday-night revelers began returning to the hostel, singing off-key and puking enthusiastically. Maybe this was poetic justice for my own faux-inebriation; if so, I was being repaid a thousandfold. I started wishing I'd shelled out for classier accommodation, but since there was no employer picking up my expenses, it was going to be hard enough dealing

with my lie to Kate without spending even more on the trip.

Forget the arithmetic of scones; I knew how to make digital currency reproduce like the marching brooms of the sorcerer's apprentice. It might even have been possible to milk the benefits without Sam noticing; I could try to hide my far-sider trading behind the manipulations of the border we used routinely to exchange messages.

I had no idea how to contain the side-effects, though. I had no idea what else such meddling would disrupt, how many people I might kill or maim in the process.

I buried my head beneath the pillows and tried to find a way to get to sleep through the noise. I ended up calculating powers of seven, a trick I hadn't used since childhood. I'd never been a prodigy at mental arithmetic, and the concentration required to push on past the easy cases drained me far faster than any physical labor. *Two hundred and eighty-two million, four hundred and seventy-five thousand, two hundred and forty-nine.* The numbers rose into the stratosphere like bean stalks, until they grew too high and tore themselves apart, leaving behind a cloud of digits drifting through my skull like black confetti.

* * * *

"The problem is under control," I told Sam. "I've located the source, and I've taken steps to prevent a recurrence."

"Are you sure of that?" As he spoke, the three-holed torus on the screen twisted restlessly. In fact I'd chosen the icon myself, and its appearance wasn't influenced by Sam at all, but it was impossible not to project emotions onto its writhing.

I said, "I'm certain that I know who was responsible for the incursion on Wednesday. It was done without malice; in fact the person who did it doesn't even realize that he crossed the border. I've modified the operating system on his computer so that it won't allow him to do the same thing again; if he tries, it will simply give him the same answers as before, but this time the calculations won't actually be performed."

"That's good to hear," Sam said. "Can you describe these calculations?"

I was as invisible to Sam as he was to me, but out of habit I tried to keep my face composed. "I don't see that as part of our agreement," I

replied.

Sam was silent for a few seconds. “That’s true, Bruno. But it might provide us with a greater sense of reassurance if we knew what caused the breach in the first place.”

I said, “I understand. But we’ve made a decision.” *We* was Alison and I; Yuen was still in hospital, in no state to do anything. Alison and I, speaking for the world.

“I’ll put your position to my colleagues,” he said. “We’re not your enemy, Bruno.” His tone sounded regretful, and these nuances *were* under his control.

“I know that,” I replied. “Nor are we yours. Yet you’ve chosen to keep most of the details of your world from us. We don’t view that as evidence of hostility, so you have no grounds to complain if we keep a few secrets of our own.”

“I’ll contact you again soon,” Sam said.

The messenger window closed. I emailed an encrypted transcript to Alison, then slumped across my desk. My head was throbbing, but the encounter really hadn’t gone too badly. Of course Sam and his colleagues would have preferred to know everything; of course they were going to be disappointed and reproachful. That didn’t mean they were going to abandon the benign policies of the last decade. The important thing was that my assurance would prove to be reliable: the incursion would not be repeated.

I had work to do, the kind that paid bills. Somehow I summoned up the discipline to push the whole subject aside and get on with a report on stochastic methods for resolving distributed programming bottlenecks that I was supposed to be writing for a company in Singapore.

Four hours later, when the doorbell rang, I’d left my desk to raid the kitchen. I didn’t bother checking the doorstep camera; I just walked down the hall and opened the door.

Campbell said, “How are you, Bruno?”

“I’m fine. Why didn’t you tell me you were coming to Sydney?”

“Aren’t you going to ask me how I found your house?”

“How?”

He held up his phone. There was a text message from me, or at least from my phone; it had SMS’d its GPS coordinates to him.

“Not bad,” I conceded.

“I believe they recently added ‘corrupting communications devices’ to the list of terrorism-related offenses in Australia. You could probably get me thrown into solitary confinement in a maximum security prison.”

“Only if you know at least ten words of Arabic.”

“Actually I spent a month in Egypt once, so anything’s possible. But I don’t think you really want to go to the police.”

I said, “Why don’t you come in?”

As I showed him to the living room my mind was racing. Maybe he’d found the relay behind the bookshelf, but surely not before I’d left his house. Had he managed to get a virus into my phone remotely? I’d thought my security was better than that.

Campbell said, “I’d like you to explain why you bugged my computer.”

“I’m growing increasingly unsure of that myself. The correct answer might be that you wanted me to.”

He snorted. “That’s rich! I admit that I deliberately allowed a rumor to start about my work, because I was curious as to why you and Alison Tierney called off your search. I wanted to see if you’d come sniffing around. As you did. But that was hardly an invitation to steal all my work.”

“What was the point of the whole exercise for you, then, if not a way of stealing something from Alison and me?”

“You can hardly compare the two. I just wanted to confirm my suspicion that you actually found something.”

“And you believe that you’ve confirmed that?”

He shook his head, but it was with amusement, not denial. I said, “Why are you here? Do you think I’m going to publish your crackpot theory as my own? I’m too old to get the Fields Medal, but maybe you think it’s

Nobel material."

"Oh, I don't think you're interested in fame. As I said, I think you beat me to the prize a long time ago."

I rose to my feet abruptly; I could feel myself scowling, my fists tightening. "So what's the bottom line? You want to press charges against me for the laptop? Go ahead. We can each get a fine *in absentia*."

Campbell said, "I want to know exactly what was so important to you that you crossed the Tasman, lied your way into my house, abused my hospitality, and stole my files. I don't think it was simply curiosity, or jealousy. I think you found something ten years ago, and now you're afraid my work is going to put it at risk."

I sat down again. The rush of adrenaline I'd experienced at being cornered had dissipated. I could almost hear Alison whispering in my ear, "Either you kill him, Bruno, or you recruit him." I had no intention of killing anyone, but I wasn't yet certain that these were the only two choices.

I said, "And if I tell you to mind your own business?"

He shrugged. "Then I'll work harder. I know you've screwed that laptop, and maybe the other computers in my house, but I'm not so broke that I can't get a new machine."

Which would be a hundred times faster. He'd re-run every search, probably with wider parameter ranges. The suitcase nuke from Sparseland that had started this whole mess would detonate again, and for all I knew it could be ten times, a hundred times, more powerful.

I said, "Have you ever wanted to join a secret society?"

Campbell gave an incredulous laugh. "No!"

"Neither did I. Too bad."

I told him everything. The discovery of the defect. Industrial Algebra's pursuit of the result. The epiphany in Shanghai. Sam establishing contact. The treaty, the ten quiet years. Then the sudden jolt of his own work, and the still-unfolding consequences.

Campbell was clearly shaken, but despite the fact that I'd confirmed his original suspicion he wasn't ready to take my word for the whole story.

I knew better than to invite him into my office for a demonstration; faking it there would have been trivial. We walked to the local shopping center, and I handed him two hundred dollars to buy a new notebook. I told him the kind of software he'd need to download, without limiting his choice to any particular package. Then I gave him some further instructions. Within half an hour, he had seen the defect for himself, and nudged the border a short distance in each direction.

We were sitting in the food hall, surrounded by boisterous teenagers who'd just got out from school. Campbell was looking at me as if I'd seized a toy machine gun from his hands, transformed it into solid metal, then bashed him over the head with it.

I said, "Cheer up. There was no war of the worlds after Shanghai; I think we're going to survive this, too." After all these years, the chance to share the burden with someone new was actually making me feel much more optimistic.

"The defect is *dynamic*," he muttered. "That changes everything."

"You don't say."

Campbell scowled. "I don't just mean the politics, the dangers. I'm talking about the underlying physical model."

"Yeah?" I hadn't come close to examining that issue seriously; it had been enough of a struggle coming to terms with his original calculations.

"All along, I've assumed that there were exact symmetries in the Planck scale physics that accounted for a stable boundary between macroscopic arithmetics. It was an artificial restriction, but I took it for granted, because anything else seemed..."

"Unbelievable?"

"Yes." He blinked and looked away, surveying the crowd of diners as if he had no idea how he'd ended up among them. "I'm flying back in a few hours."

"Does Bridget know why you came?"

"Not exactly."

I said, “No one else can know what I’ve told you. Not yet. The risks are too great, everything’s too fluid.”

“Yeah.” He met my gaze. He wasn’t just humoring me; he understood what people like IA might do.

“In the long term,” I said, “we’re going to have to find a way to make this safe. To make everyone safe.” I’d never quite articulated that goal before, but I was only just beginning to absorb the ramifications of Campbell’s insights.

“How?” he wondered. “Do we want to build a wall, or do we want to tear one down?”

“I don’t know. The first thing we need is a better map, a better feel for the whole territory.”

He’d hired a car at the airport in order to drive here and confront me; it was parked in a side street close to my house. I walked him to it.

We shook hands before parting. I said, “Welcome to the reluctant cabal.”

Campbell winced. “Let’s find a way to change it from reluctant to redundant.”

* * * *

In the weeks that followed, Campbell worked on refinements to his theory, emailing Alison and me every few days. Alison had taken my unilateral decision to recruit Campbell with much more equanimity than I’d expected. “Better to have him inside the tent,” was all she’d said.

This proved to be an understatement. While the two of us soon caught up with him on all the technicalities, it was clear that his intuition on the subject, hard-won over many years of trial and error, was the key to his spectacular progress now. Merely stealing his notes and his algorithms would never have brought us so far.

Gradually, the dynamic version of the theory took shape. As far as macroscopic objects were concerned—and in this context, “macroscopic” stretched all the way down to the quantum states of subatomic particles—all traces of Platonic mathematics were banished. A “proof” concerning the integers was just a class of physical processes, and the

result of that proof was neither read from, nor written to, any universal book of truths. Rather, the agreement between proofs was simply a strong, but imperfect, correlation between the different processes that counted as proofs of the same thing. Those correlations arose from the way that the primordial states of Planck-scale physics were carved up—imperfectly—into subsystems that appeared to be distinct objects.

The truths of mathematics *appeared* to be enduring and universal because they persisted with great efficiency within the states of matter and space-time. But there was a built-in flaw in the whole idealization of distinct objects, and the point where the concept finally cracked open was the defect Alison and I had found in our volunteers' data, which appeared to any macroscopic test as the border between contradictory mathematical systems.

We'd derived a crude empirical rule which said that the border shifted when a proposition's neighbors outvoted it. If you managed to prove that x+1=y+1 and x-1=y-1, then x=y became a sitting duck, even if it hadn't been true before. The consequences of Campbell's search had shown that the reality was more complex, and in his new model, the old border rule became an approximation for a more subtle process, anchored in the dynamics of primordial states that knew nothing of the arithmetic of electrons and apples. The near-side arithmetic Campbell had blasted into the far side hadn't got there by besieging the target with syllogisms; it had got there because he'd gone straight for a far deeper failure in the whole idea of "integers" than Alison and I had ever dreamed of.

Had Sam dreamed of it? I waited for his next contact, but as the weeks passed he remained silent, and the last thing I felt like doing was calling him myself. I had enough people to lie to without adding him to the list.

Kate asked me how work was going, and I waffled about the details of the three uninspiring contracts I'd started recently. When I stopped talking, she looked at me as if I'd just stammered my way through an unconvincing denial of some unspoken crime. I wondered how my mixture of concealed elation and fear was coming across to her. Was that how the most passionate, conflicted adulterer would appear? I didn't actually reach the brink of confession, but I pictured myself approaching it. I had less reason now to think that the secret would bring her harm than when I'd first made my decision to keep her in the dark. But then, what if I told her everything, and the next day Campbell was kidnapped and tortured? If we were all being watched, and the people doing it were good at their jobs, we'd only know about it when it was too late.

Campbell's emails dropped off for a while, and I assumed he'd hit a roadblock. Sam had offered no further complaints. Perhaps, I thought, this was the new *status quo*, the start of another quiet decade. I could live with that.

Then Campbell flung his second grenade. He reached me by IM and said, "I've started making maps."

"Of the defect?" I replied.

"Of the planets."

I stared at his image, uncomprehending.

"The far-side planets," he said. "*The physical worlds*."

He'd bought himself some time on a geographically scattered set of processor clusters. He was no longer repeating his dangerous incursions, of course, but by playing around in the natural ebb and flow at the border, he'd made some extraordinary discoveries.

Alison and I had realized long ago that random "proofs" in the natural world would influence what happened at the border, but Campbell's theory made that notion more precise. By looking at the exact timing of changes to propositions at the border, measured in a dozen different computers world-wide, he had set up a kind of ... radar? CT machine? Whatever you called it, it allowed him to deduce the locations where the relevant natural processes were occurring, and his model allowed him to distinguish between both near-side and far-side processes, and processes in matter and those in vacuum. He could measure the density of far-side matter out to a distance of several light-hours, and crudely image nearby planets.

"Not just on the far side," he said. "I validated the technique by imaging our own planets." He sent me a data log, with comparisons to an online almanac. For Jupiter, the farthest of the planets he'd located, the positions were out by as much as a hundred thousand kilometers; not exactly GPS quality, but that was a bit like complaining that your abacus couldn't tell north from north-west.

"Maybe that's how Sam found us in Shanghai?" I wondered. "The same kind of thing, only more refined?"

Campbell said, "Possibly."

“So what about the far-side planets?”

“Well, here’s the first interesting thing. None of the planets coincide with ours. Nor does their sun with our sun.” He sent me an image of the far-side system, one star and its six planets, overlaid on our own.

“But Sam’s time lags,” I protested, “when we communicate—”

“Make no sense if he’s too far away. Exactly. So he is *not* living on any of these planets, and he’s not even in a natural orbit around their star. He’s in powered flight, moving with the Earth. Which suggests to me that they’ve known about us for much longer than Shanghai.”

“Known about us,” I said, “but maybe they still didn’t anticipate anything like Shanghai.” When we’d set Luminous on to the task of eliminating the defect—not knowing that we were threatening anyone—it had taken several minutes before the far side had responded. Computers on board a spacecraft moving with the Earth would have detected the assault quickly, but it might have taken the recruitment of larger, planet-bound machines, minutes away at lightspeed, to repel it.

Until I’d encountered Campbell’s theories, my working assumption had been that Sam’s world was like a hidden message encoded in the Earth, with the different arithmetic giving different meanings to all the air, water, and rock around us. But their matter was not bound to our matter; they didn’t need our specks of dust or molecules of air to represent the dark integers. The two worlds split apart at a much lower level; vacuum could be rock, and rock, vacuum.

I said, “So do you want the Nobel for physics, or peace?”

Campbell smiled modestly. “Can I hold out for both?”

“That’s the answer I was looking for.” I couldn’t get the stupid Cold War metaphors out of my brain: what would Sam’s hotheaded colleagues think, if they knew that we were now flying spy planes over their territory? Saying “screw them, they were doing it first!” might have been a fair response, but it was not a particularly helpful one.

I said, “We’re never going to match their Sputnik, unless you happen to know a trustworthy billionaire who wants to help us launch a space probe on a very strange trajectory. Everything we want to do has to work from Earth.”

“I’ll tear up my letter to Richard Branson then, shall I?”

I stared at the map of the far-side solar system. “There must be some relative motion between their star and ours. It can’t have been this close for all that long.”

“I don’t have enough accuracy in my measurements to make a meaningful estimate of the velocity,” Campbell said. “But I’ve done some crude estimates of the distances between their stars, and it’s much smaller than ours. So it’s not all that unlikely to find *some* star this close to us, even if it’s unlikely to be the same one that was close a thousand years ago. Then again, there might be a selection effect at work here: the whole reason Sam’s civilization managed to notice us at all was *because* we weren’t shooting past them at a substantial fraction of lightspeed.”

“Okay. So *maybe* this is their home system, but it could just as easily be an expeditionary base for a team that’s been following our sun for thousands of years.”

“Yes.”

I said, “Where do we go with this?”

“I can’t increase the resolution much,” Campbell replied, “without buying time on a lot more clusters.” It wasn’t that he needed much processing power for the calculations, but there were minimum prices to be paid to do anything at all, and what would give us clearer pictures would be more computers, not more time on each one.

I said, “We can’t risk asking for volunteers, like the old days. We’d have to lie about what the download was for, and you can be certain that somebody would reverse-engineer it and catch us out.”

“Absolutely.”

I slept on the problem, then woke with an idea at four A.M. and went to my office, trying to flesh out the details before Campbell responded to my email. He was bleary-eyed when the messenger window opened; it was later in Wellington than in Sydney, but it looked as if he’d had as little sleep as I had.

I said, “We use the internet.”

“I thought we decided that was too risky.”

“Not screensavers for volunteers; I’m talking about *the internet itself*. We work out a way to do the calculations using nothing but data packets and network routers. We bounce traffic all around the world, and we get the geographical resolution for free.”

“You’ve got to be joking, Bruno—”

“Why? *Any* computing circuit can be built by stringing together enough NAND gates; you think we can’t leverage packet switching into a NAND gate? But that’s just the proof that it’s possible; I expect we can actually make it a thousand times tighter.”

Campbell said, “I’m going to get some aspirin and come back.”

We roped in Alison to help, but it still took us six weeks to get a workable design, and another month to get it functioning. We ended up exploiting authentication and error-correction protocols built into the internet at several different layers; the heterogeneous approach not only helped us do all the calculations we needed, but made our gentle siphoning of computing power less likely to be detected and mistaken for anything malicious. In fact we were “stealing” far less from the routers and servers of the net than if we’d sat down for a hardcore 3D multiplayer gaming session, but security systems had their own ideas about what constituted fair use and what was suspicious. The most important thing was not the size of the burden we imposed, but the signature of our behavior.

Our new globe-spanning arithmetical telescope generated pictures far sharper than before, with kilometer-scale resolution out to a billion kilometers. This gave us crude relief-maps of the far-side planets, revealing mountains on four of them, and what might have been oceans on two of those four. If there were any artificial structures, they were either too small to see, or too subtle in their artificiality.

The relative motion of our sun and the star these planets orbited turned out to be about six kilometers per second. In the decade since Shanghai, the two solar systems had changed their relative location by about two billion kilometers. Wherever the computers were now that had fought with Luminous to control the border, they certainly hadn’t been on any of these planets at the time. Perhaps there were two ships, with one following the Earth, and the other, heavier one saving fuel by merely following the sun.

Yuen had finally recovered his health, and the full cabal held an IM-conference to discuss these results.

“We should be showing these to geologists, xenobiologists ... everyone,” Yuen lamented. He wasn’t making a serious proposal, but I shared his sense of frustration.

Alison said, “What I regret most is that we can’t rub Sam’s face in these pictures, just to show him that we’re not as stupid as he thinks.”

“I imagine his own pictures are sharper,” Campbell replied.

“Which is as you’d expect,” Alison retorted, “given a head start of a few centuries. If they’re so brilliant on the far side, why do they need *us* to tell them what you did to jump the border?”

“They might have guessed precisely what I did,” he countered, “but they could still be seeking confirmation. Perhaps what they really want is to rule out the possibility that we’ve discovered something different, something they’ve never even thought of.”

I gazed at the false colors of one contoured sphere, imagining gray-blue oceans, snow-topped mountains with alien forests, strange cities, wondrous machines. Even if that was pure fantasy and this temporary neighbor was barren, there had to be a living homeworld from which the ships that pursued us had been launched.

After Shanghai, Sam and his colleagues had chosen to keep us in the dark for ten years, but it had been our own decision to cement the mistrust by holding on to the secret of our accidental weapon. If they’d already guessed its nature, then they might already have found a defense against it, in which case our silence bought us no advantage at all to compensate for the suspicion it engendered.

If that assumption was wrong, though? Then handing over the details of Campbell’s work could be just what the far-side hawks were waiting for, before raising their shields and crushing us.

I said, “We need to make some plans. I want to stay hopeful, I want to keep looking for the best way forward, but we need to be prepared for the worst.”

* * * *

Transforming that suggestion into something concrete required far more work than I'd imagined; it was three months before the pieces started coming together. When I finally shifted my gaze back to the everyday world, I decided that I'd earned a break. Kate had a free weekend approaching; I suggested a day in the Blue Mountains.

Her initial response was sarcastic, but when I persisted she softened a little, and finally agreed.

On the drive out of the city, the chill that had developed between us slowly began to thaw. We played JJJ on the car radio—laughing with disbelief as we realized that today's cutting-edge music consisted mostly of cover versions and re-samplings of songs that had been hits when we were in our twenties—and resurrected old running jokes from the time when we'd first met.

As we wound our way into the mountains, though, it proved impossible simply to turn back the clock. Kate said, "Whoever you've been working for these last few months, can you put them on your blacklist?"

I laughed. "That will scare them." I switched to my best Brando voice. "You're on Bruno Costanzo's blacklist. You'll never run distributed software efficiently in this town again."

She said, "I'm serious. I don't know what's so stressful about the work, or the people, but it's really screwing you up."

I could have made her a promise, but it would have been hard enough to sound sincere as I spoke the words, let alone live up to them. I said, "Beggars can't be choosers."

She shook her head, her mouth tensed in frustration. "If you really want a heart attack, fine. But don't pretend that it's all about money. We're never that broke, and we're never that rich. Unless it's all going into your account in Zurich."

It took me a few seconds to convince myself that this was nothing more than a throwaway reference to Swiss banks. Kate knew about Alison, knew that we'd once been close, knew that we still kept in touch. She had plenty of male friends from her own past, and they all lived in Sydney; for more than five years, Alison and I hadn't even set foot on the same continent.

We parked the car, then walked along a scenic trail for an hour,

mostly in silence. We found a spot by a stream, with tiered rocks smoothed by some ancient river, and ate the lunch I'd packed.

Looking out into the blue haze of the densely wooded valley below, I couldn't keep the image of the crowded skies of the far side from my mind. A dazzling richness surrounded us: alien worlds, alien life, alien culture. There had to be a way to end our mutual suspicion, and work toward a genuine exchange of knowledge.

As we started back toward the car, I turned to Kate. "I know I've neglected you," I said. "I've been through a rough patch, but everything's going to change. I'm going to make things right."

I was prepared for a withering rebuff, but for a long time she was silent. Then she nodded slightly and said, "Okay."

As she reached across and took my hand, my wrist began vibrating. I'd buckled to the pressure and bought a watch that shackled me to the net twenty-four hours a day.

I freed my hand from Kate's and lifted the watch to my face. The bandwidth reaching me out in the sticks wasn't enough for video, but a stored snapshot of Alison appeared on the screen.

"This is for *emergencies only*," I snarled.

"Check out a news feed," she replied. The acoustics were focused on my ears; Kate would get nothing but the bad-hearing-aid-at-a-party impression that made so many people want to punch their fellow commuters on trains.

"Why don't you just summarize whatever it is I'm meant to have noticed?"

Financial computing systems were going haywire, to an extent that was already being described as terrorism. Most trading was closed for the weekend, but some experts were predicting the crash of the century, come Monday.

I wondered if the cabal itself was to blame; if we'd inadvertently corrupted the whole internet by coupling its behavior to the defect. That was nonsense, though. Half the transactions being garbled were taking place on secure, interbank networks that shared no hardware with our global computer. This was coming from the far side.

“Have you contacted Sam?” I asked her.

“I can’t raise him.”

“Where are you going?” Kate shouted angrily. I’d unconsciously broken into a jog; I wanted to get back to the car, back to the city, back to my office.

I stopped and turned to her. “Run with me? Please? This is important.”

“You’re joking! I’ve spent half a day hiking, I’m not running anywhere!”

I hesitated, fantasizing for a moment that I could sit beneath a gum tree and orchestrate everything with my Dick Tracy watch before its battery went flat.

I said, “You’d better call a taxi when you get to the road.”

“You’re taking the car?” Kate stared at me, incredulous. “You piece of shit!”

“I’m sorry.” I tossed my backpack on the ground and started sprinting.

“We need to deploy,” I told Alison.

“I know,” she said. “We’ve already started.”

It was the right decision, but hearing it still loosened my bowels far more than the realization that the far side were attacking us. Whatever their motives, at least they were unlikely to do more harm than they intended. I was much less confident about our own abilities.

“Keep trying to reach Sam,” I insisted. “This is a thousand times more useful if they know about it.”

Alison said, “I guess this isn’t the time for *Dr. Strangelove* jokes.”

Over the last three months, we’d worked out a way to augment our internet “telescope” software to launch a barrage of Campbell-style attacks on far-side propositions if it saw our own mathematics being encroached upon. The software couldn’t protect the whole border, but there were millions of individual trigger points, forming a randomly shifting minefield.

The plan had been to buy ourselves some security, without ever reaching the point of actual retaliation. We'd been waiting to complete a final round of tests before unleashing this version live on the net, but it would only take a matter of minutes to get it up and running.

"Anything being hit besides financials?" I asked.

"Not that I'm picking up."

If the far side was deliberately targeting the markets, that was infinitely preferable to the alternative: that financial systems had simply been the most fragile objects in the path of a much broader assault. Most modern engineering and aeronautical systems were more interested in resorting to fall-backs than agonizing over their failures. A bank's computer might declare itself irretrievably compromised and shut down completely, the instant certain totals failed to reconcile; those in a chemical plant or an airliner would be designed to fail more gracefully, trying simpler alternatives and bringing all available humans into the loop.

I said, "Yuen and Tim—?"

"Both on board," Alison confirmed. "Monitoring the deployment, ready to tweak the software if necessary."

"Good. You really won't need me at all, then, will you?"

Alison's reply dissolved into digital noise, and the connection cut out. I refused to read anything sinister into that; given my location, I was lucky to have any coverage at all. I ran faster, trying not to think about the time in Shanghai when Sam had taken a mathematical scalpel to all of our brains. Luminous had been screaming out our position like a beacon; we would not be so easy to locate this time. Still, with a cruder approach, the hawks could take a hatchet to everyone's head. *Would they go that far?* Only if this was meant as much more than a threat, much more than intimidation to make us hand over Campbell's algorithm. Only if this was the end game: no warning, no negotiations, just Sparseland wiped off the map forever.

Fifteen minutes after Alison's call, I reached the car. Apart from the entertainment console it didn't contain a single microchip; I remembered the salesman laughing when I'd queried that twice. "What are you afraid of? Y3K?" The engine started immediately.

I had an ancient secondhand laptop in the trunk; I put it beside me on the passenger seat and started booting it up while I drove out on to the

access road, heading for the highway. Alison and I had worked for a fortnight on a stripped-down operating system, as simple and robust as possible, to run on these old computers; if the far side kept reaching down from the arithmetic stratosphere, these would be like concrete bunkers compared to the glass skyscrapers of more modern machines. The four of us would also be running different versions of the OS, on CPUs with different instruction sets; our bunkers were scattered mathematically as well as geographically.

As I drove on to the highway, my watch stuttered back to life. Alison said, “Bruno? Can you hear me?”

“Go ahead.”

“Three passenger jets have crashed,” she said. “Poland, Indonesia, South Africa.”

I was dazed. Ten years before, when I’d tried to bulldoze his whole mathematical world into the sea, Sam had spared my life. Now the far side was slaughtering innocents.

“Is our minefield up?”

“It’s been up for ten minutes, but nothing’s tripped it yet.”

“You think they’re steering through it?”

Alison hesitated. “I don’t see how. There’s no way to predict a safe path.” We were using a quantum noise server to randomize the propositions we tested.

I said, “We should trigger it manually. One counter-strike to start with, to give them something to think about.” I was still hoping that the downed jets were unintended, but we had no choice but to retaliate.

“Yeah.” Alison’s image was live now; I saw her reach down for her mouse. She said, “It’s not responding. The net’s too degraded.” All the fancy algorithms that the routers used, and that we’d leveraged so successfully for our imaging software, were turning them into paperweights. The internet was robust against high levels of transmission noise and the loss of thousands of connections, but not against the decay of arithmetic itself.

My watch went dead. I looked to the laptop; it was still working. I

reached over and hit a single hotkey, launching a program that would try to reach Alison and the others the same way we'd talked to Sam: by modulating part of the border. In theory, the hawks might have moved the whole border—in which case we were screwed—but the border was vast, and it made more sense for them to target their computing resources on the specific needs of the assault itself.

A small icon appeared on the laptop's screen, a single letter A in reversed monochrome. I said, "Is this working?"

"Yes," Alison replied. The icon blinked out, then came back again. We were doing a Hedy Lamarr, hopping rapidly over a predetermined sequence of border points to minimize the chance of detection. Some of those points would be missing, but it looked as if enough of them remained intact.

The A was joined by a Y and a T. The whole cabal was online now, whatever that was worth. What we needed was S, but S was not answering.

Campbell said grimly, "I heard about the planes. I've started an attack." The tactic we had agreed upon was to take turns running different variants of Campbell's border-jumping algorithm from our scattered machines.

I said, "The miracle is that they're not hitting us the same way we're hitting them. They're just pushing down part of the border with the old voting method, step by step. If we'd given them what they'd asked for, we'd all be dead by now."

"Maybe not," Yuen replied. "I'm only halfway through a proof, but I'm 90 percent sure that Tim's method is asymmetrical. It only works in one direction. Even if we'd told them about it, they couldn't have turned it against us."

I opened my mouth to argue, but if Yuen was right that made perfect sense. The far side had probably been working on the same branch of mathematics for centuries; if there had been an equivalent weapon that could be used from their vantage point, they would have discovered it long ago.

My machine had synchronized with Campbell's, and it took over the assault automatically. We had no real idea what we were hitting, except that the propositions were further from the border, describing far simpler arithmetic on the dark integers than anything of ours that the far side had yet

touched. *Were we crippling machines? Taking lives?* I was torn between a triumphant vision of retribution, and a sense of shame that we'd allowed it to come to this.

Every hundred meters or so, I passed another car sitting motionless by the side of the highway. I was far from the only person still driving, but I had a feeling Kate wouldn't have much luck getting a taxi. She had water in her backpack, and there was a small shelter at the spot where we'd parked. There was little to be gained by reaching my office now; the laptop could do everything that mattered, and I could run it from the car battery if necessary. If I turned around and went back for Kate, though, I'd have so much explaining to do that there'd be no time for anything else.

I switched on the car radio, but either its digital signal processor was too sophisticated for its own good, or all the local stations were out.

"Anyone still getting news?" I asked.

"I still have radio," Campbell replied. "No TV, no internet. Landlines and mobiles here are dead." It was the same for Alison and Yuen. There'd been no more reports of disasters on the radio, but the stations were probably as isolated now as their listeners. Ham operators would still be calling each other, but journalists and newsrooms would not be in the loop. I didn't want to think about the contingency plans that might have been in place, given ten years' preparation and an informed population.

By the time I reached Penrith there were so many abandoned cars that the remaining traffic was almost gridlocked. I decided not to even try to reach home. I didn't know if Sam had literally scanned my brain in Shanghai and used that to target what he'd done to me then, and whether or not he could use the same neuroanatomical information against me now, wherever I was, but staying away from my usual haunts seemed like one more small advantage to cling to.

I found a gas station, and it was giving priority to customers with functioning cars over hoarders who'd appeared on foot with empty cans. Their EFTPOS wasn't working, but I had enough cash for the gas and some chocolate bars.

As dusk fell the streetlights came on; the traffic lights had never stopped working. All four laptops were holding up, hurling their grenades into the far side. The closer the attack front came to simple arithmetic, the more resistance it would face from natural processes voting at the border for near-side results. Our enemy had their supercomputers; we had every

atom of the Earth, following its billion-year-old version of the truth.

We had modeled this scenario. The sheer arithmetical inertia of all that matter would buy us time, but in the long run a coherent, sustained, computational attack could still force its way through.

How would we die? Losing consciousness first, feeling no pain? Or was the brain more robust than that? Would all the cells of our bodies start committing apoptosis, once their biochemical errors mounted up beyond repair? Maybe it would be just like radiation sickness. We'd be burned by decaying arithmetic, just as if it was nuclear fire.

My laptop beeped. I swerved off the road and parked on a stretch of concrete beside a dark shopfront. A new icon had appeared on the screen: the letter S.

Sam said, "Bruno, this was not my decision."

"I believe you," I said. "But if you're just a messenger now, what's your message?"

"If you give us what we asked for, we'll stop the attack."

"We're hurting you, aren't we?"

"We know we're hurting *you*," Sam replied. Point taken: we were guessing, firing blind. He didn't have to ask about the damage we'd suffered.

I steeled myself, and followed the script the cabal had agreed upon. "We'll give you the algorithm, but only if you retreat back to the old border, and then seal it."

Sam was silent for four long heartbeats.

"Seal it?"

"I think you know what I mean." In Shanghai, when we'd used Luminous to try to ensure that Industrial Algebra could not exploit the defect, we'd contemplated trying to seal the border rather than eliminating the defect altogether. The voting effect could only shift the border if it was crinkled in such a way that propositions on one side could be outnumbered by those on the other side. It was possible—given enough time and computing power—to smooth the border, to iron it flat. Once that was done,

everywhere, the whole thing would become immovable. No force in the universe could shift it again.

Sam said, “You want to leave us with no weapon against you, while you still have the power to harm us.”

“We won’t have that power for long. Once you know exactly what we’re using, you’ll find a way to block it.”

There was a long pause. Then, “Stop your attacks on us, and we’ll consider your proposal.”

“We’ll stop our attacks when you pull the border back to the point where our lives are no longer at risk.”

“How would you even know that we’ve done that?” Sam replied. I wasn’t sure if the condescension was in his tone or just his words, but either way I welcomed it. The lower the far side’s opinion of our abilities, the more attractive the deal became for them.

I said, “Then you’d better back up far enough for all our communications systems to recover. When I can get news reports and see that there are no more planes going down, no power plants exploding, then we’ll start the ceasefire.”

Silence again, stretching out beyond mere hesitancy. His icon was still there, though, the S unblinking. I clutched at my shoulder, hoping that the burning pain was just tension in the muscle.

Finally: “All right. We agree. We’ll start shifting the border.”

I drove around looking for an all-night convenience store that might have had an old analog TV sitting in a corner to keep the cashier awake—that seemed like a good bet to start working long before the wireless connection to my laptop—but Campbell beat me to it. New Zealand radio and TV were reporting that the “digital blackout” appeared to be lifting, and ten minutes later Alison announced that she had internet access. A lot of the major servers were still down, or their sites weirdly garbled, but Reuters was starting to post updates on the crisis.

Sam had kept his word, so we halted the counter-strikes. Alison read from the Reuters site as the news came in. Seventeen planes had crashed, and four trains. There’d been fatalities at an oil refinery, and half a dozen manufacturing plants. One analyst put the global death toll at five thousand

and rising.

I muted the microphone on my laptop and spent thirty seconds shouting obscenities and punching the dashboard. Then I rejoined the cabal.

Yuen said, “I’ve been reviewing my notes. If my instinct is worth anything, the theorem I mentioned before is correct: if the border is sealed, they’ll have no way to touch us.”

“What about the upside for them?” Alison asked. “Do you think they can protect themselves against Tim’s algorithm, once they understand it?”

Yuen hesitated. “Yes and no. Any cluster of near-side truth values it injects into the far side will have a non-smooth border, so they’ll be able to remove it with sheer computing power. In that sense, they’ll never be defenseless. But I don’t see how there’s anything they can do to prevent the attacks in the first place.”

“Short of wiping us out,” Campbell said.

I heard an infant sobbing. Alison said, “That’s Laura. I’m alone here. Give me five minutes.”

I buried my head in my arms. I still had no idea what the right course would have been. If we’d handed over Campbell’s algorithm immediately, might the good will that bought us have averted the war? Or would the same attack merely have come sooner? What criminal vanity had ever made the three of us think we could shoulder this responsibility on our own? Five thousand people were dead. The hawks who had taken over on the far side would weigh up our offer, and decide that they had no choice but to fight on.

And if the reluctant cabal had passed its burden to Canberra, to Zurich, to Beijing? Would there really have been peace? Or was I just wishing that there had been more hands steeped in the same blood, to share the guilt around?

The idea came from nowhere, sweeping away every other thought. I said, “Is there any reason why the far side has to stay *connected*?”

“Connected to what?” Campbell asked.

“Connected to itself. Connected topologically. They should be able to

send down a spike, then withdraw it, but leave behind a bubble of altered truth values: a kind of outpost, sitting within the near side, with a perfect, smooth border making it impregnable. Right?"

Yuen said, "Perhaps. With both sides collaborating on the construction, that might be possible."

"Then the question is, can we find a place where we can do that so that it kills off the chance to use Tim's method completely—without crippling any process that we need just to survive?"

"*Fuck you*, Bruno!" Campbell exclaimed happily. "We give them one small Achilles tendon to slice ... and then they've got nothing to fear from us!"

Yuen said, "A watertight proof of something like that is going to take weeks, months."

"Then we'd better start work. And we'd better feed Sam the first plausible conjecture we get, so they can use their own resources to help us with the proof."

Alison came back online and greeted the suggestion with cautious approval. I drove around until I found a quiet coffee shop. Electronic banking still wasn't working, and I had no cash left, but the waiter agreed to take my credit card number and a signed authority for a deduction of one hundred dollars; whatever I didn't eat and drink would be his tip.

I sat in the café, blanking out the world, steeping myself in the mathematics. Sometimes the four of us worked on separate tasks; sometimes we paired up, dragging each other out of dead ends and ruts. There were an infinite number of variations that could be made to Campbell's algorithm, but hour by hour we whittled away at the concept, finding the common ground that no version of the weapon could do without.

By four in the morning, we had a strong conjecture. I called Sam, and explained what we were hoping to achieve.

He said, "This is a good idea. We'll consider it."

The café closed. I sat in the car for a while, drained and numb, then I called Kate to find out where she was. A couple had given her a lift almost as far as Penrith, and when their car failed she'd walked the rest of the way home.

* * * *

For close to four days, I spent most of my waking hours just sitting at my desk, watching as a wave of red inched its way across a map of the defect. The change of hue was not being rendered lightly; before each pixel turned red, twelve separate computers needed to confirm that the region of the border it represented was flat.

On the fifth day, Sam shut off his computers and allowed us to mount an attack from our side on the narrow corridor linking the bulk of the far side with the small enclave that now surrounded our Achilles' Heel. We wouldn't have suffered any real loss of essential arithmetic if this slender thread had remained, but keeping the corridor both small and impregnable had turned out to be impossible. The original plan was the only route to finality: to seal the border perfectly, the far side proper could not remain linked to its offshoot.

In the next stage, the two sides worked together to seal the enclave completely, polishing the scar where its umbilical had been sheared away. When that task was complete, the map showed it as a single burnished ruby. No known process could reshape it now. Campbell's method could have breached its border without touching it, reaching inside to reclaim it from within—but Campbell's method was exactly what this jewel ruled out.

At the other end of the vanished umbilical, Sam's machines set to work smoothing away the blemish. By early evening that, too, was done.

Only one tiny flaw in the border remained now: the handful of propositions that enabled communication between the two sides. The cabal had debated the fate of this for hours. So long as this small wrinkle persisted, in principle it could be used to unravel everything, to mobilize the entire border again. It was true that, compared to the border as a whole, it would be relatively easy to monitor and defend such a small site, but a sustained burst of brute-force computing from either side could still overpower any resistance and exploit it.

In the end, Sam's political masters had made the decision for us. What they had always aspired to was certainty, and even if their strength favored them, this wasn't a gamble they were prepared to take.

I said, "Good luck with the future."

"Good luck to Sparseland," Sam replied. I believed he'd tried to hold

out against the hawks, but I'd never been certain of his friendship. When his icon faded from my screen, I felt more relief than regret.

I'd learned the hard way not to assume that anything was permanent. Perhaps in a thousand years, someone would discover that Campbell's model was just an approximation to something deeper, and find a way to fracture these allegedly perfect walls. With any luck, by then both sides might also be better prepared to find a way to co-exist.

I found Kate sitting in the kitchen. I said, "I can answer your questions now, if that's what you want." On the morning after the disaster, I'd promised her this time would come—within weeks, not months—and she'd agreed to stay with me until it did.

She thought for a while.

"Did you have something to do with what happened last week?"

"Yes."

"Are you saying you unleashed the virus? You're the terrorist they're looking for?" To my great relief, she asked this in roughly the tone she might have used if I'd claimed to be Genghis Khan.

"No, I'm not the cause of what happened. It was my job to try and stop it, and I failed. But it wasn't any kind of computer virus."

She searched my face. "What was it, then? Can you explain that to me?"

"It's a long story."

"I don't care. We've got all night."

I said, "It started in university. With an idea of Alison's. One brilliant, beautiful, crazy idea."

Kate looked away, her face flushing, as if I'd said something deliberately humiliating. She knew I was not a mass murderer. But there were other things about me of which she was less sure.

"The story starts with Alison," I said. "But it ends here, with you."